Astronomy & Astrophysics manuscript no. gw˙detector˙paper˙v14
September 19, 2011

On the Possibility of Using Edge-on Binary Star Systems to Detect Gravitational Waves

T. N. Ukwatta[1] and T. Wickramasinghe[2]

[1] Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA.
e-mail: tilan.ukwatta@gmail.com
[2] Department of Physics, The College of New Jersey, Ewing, NJ 08628, USA.

Preprint online version: September 19, 2011

ABSTRACT

A major problem that prohibits direct detection of gravitational waves (GW) is their weakness in strength. The signal strength of a gravitational wave is inversely proportional to its source distance. Most GW sources lie very far away from Earth and by the time they reach Earth they are very weak and difficult to be detected. One way to circumvent this problem is to look at electromagnetic signatures close to the GW source itself. In this paper, we propose to detect gravitational waves using Doppler shifts of stars in the neighborhood of a GW source. In order to illustrate our idea we present a simple scenario of two identical binary stars orbiting each other, oriented edge-on in our field of view. A gravitational wave hits the system face-on and perturbs the system in such a way that the line-of-sight velocities of the two stars oscillate relative to the observer and create potentially detectable Doppler shifts in the stellar electromagnetic spectrum. In addition, our analysis has shown that GW with frequency close to the resonance frequency of the binary system may unbind the binary system.

Key words. gravitational waves

1. Introduction

After decades of extensive search, gravitational waves (GW) still elude direct detection. Einstein (1916) first predicted the theoretical existence of gravitational waves. However, the only indirect evidence thus far for their presence came from the binary pulsar, PSR 1913+16, which shows an orbital decay rate consistent with the predicted rate based on general relativity (Taylor & Weisberg, 1989). The signal strength or the strain, h, of a gravitational wave is inversely proportional to its source distance, r. Most of these GW sources lie far away from Earth and by the time the emitted waves reach Earth they are very weak in amplitude and become difficult to be detected.

Currently, there are two types of well studied GW detection methods in use (Ni, 2010). The first type is called resonant type, where various techniques are used to enhance the GW induce resonances in detectors. The second type, called free-particle type, measures distance changes between test bodies using electromagnetic waves (Freise & Strain, 2010). Both these detection methods suffer from weak GW signal strength. A significantly different method was proposed by Pyne et al. (1996), which involves measuring apparent proper motion of far away astrophysical sources. The idea is that if the period of the gravitational wave is far longer than the observing time, then the signature of the passing GW can potentially be embedded in the apparent positions of these far away sources. Gwinn et al. (1997) used this method on quasars to constrain gravitational waves in ultra-low frequency bands (10 fHz - 300 pHz).

One possible method to overcome the signal strength issue is to look for electromagnetic signatures relatively close to a GW source. Of course one has to know possible locations of GW sources to utilize this method. There are a number of candidate systems in our galaxy where gravitational wave emissions may be expected. For example, in systems such as globular clusters stars are packed so tightly that there is a fair chance for the existence of rotating black hole pairs, which could emit gravitational waves.

Extra-solar planet searches have successfully utilized line-of-sight velocity measurements to find planets in other solar systems. They are continually improving the capabilities of their spectrographs and today's modern spectroscopes are capable of detecting line-of-sight velocity variations in stars down to ~ 1 m/s (Mayor et al., 2009). This corresponds to a limiting Doppler shift, z, of $\sim 10^{-8}$ ($z \approx v/c$ for $v \ll c$; here v is the line-of-sight velocity of the star and c is the speed of light).

Extra-solar planet searches use the fact that a star with a planet will wobble about its center of mass creating a detectable Doppler shift in its spectral lines. However, the gravitational effect of a hidden planet is not the only phenomenon that could create Doppler shifts. A gravitational wave that passes through a star system can potentially create detectable Doppler shifts. In this paper, we propose to use Doppler effects of edge-on binary star systems in the close vicinity of these candidate sources to detect gravitational waves. In §2 we illustrate our idea using an edge-on binary star system with identical masses. Then in §3, we discuss potential implications of GW interaction with binary systems. Finally, we summarize briefly our main conclusions in §4.

2. Binary-Star GW Detector

2.1. Binary system as a driven harmonic oscillator

In order to illustrate our idea, we consider a simple system of two identical stars orbiting each other, oriented edge-on in our field-of-view as shown in Fig. 1. A gravitational wave of single polarization hits the system face-on and perturbs it in such a way that line-of-sight velocities of its stars oscillate relative to the

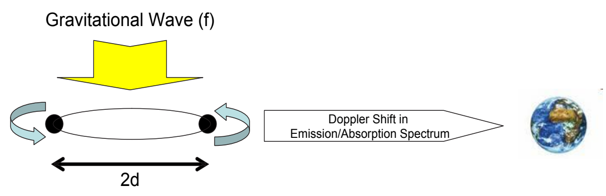

Fig. 1. Binary star system with identical masses orbiting each other edge-on from the point-of-view from Earth.

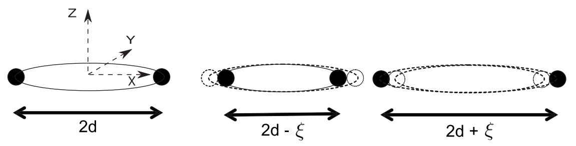

Fig. 2. Oscillation of the system as a gravitational wave passes.

observer and creates Doppler shifts in the stellar electromagnetic spectrum. Notice that this method too belongs to the resonant detection category as the binary system itself resonates to the incoming GW.

Let $2\,d$ be the separation of two identical stars of mass m and the rotational period of the system be larger than the period of the passing gravitational wave. When a gravitational wave with a wavelength comparable to the separation of the two stars passes through the system, it will distort the local space-time such that the two stars will oscillate as shown in Fig. 2. This oscillation will create alternating redshifts and blueshifts in both stars. However, note that at a given time when one star shows a redshift then the other star will show a blueshift.

If the above binary system is stretched by a small length, $\xi(t)$, due to a passing GW, the system behaves just like a simple detector and can be described by the following differential equation.

$$\ddot{\xi} + \alpha\dot{\xi} + \omega_0^2 \xi = \lambda \tag{1}$$

Here λ is the tidal acceleration due to the GW. The coefficients α and ω_0 are the damping coefficient and the natural frequency respectively of the system. These coefficients arise due to the mutual gravity between the two stars and to the fact that the system is in the potential well of external stars. The system is subject to various tidal forces exerted by the two stars themselves and by the stars in the close proximity (Binney & Tremaine, 2008). The stars in this case do not move in a Kepler potential and the orbits will not be closed, i.e., radial and azimuthal periods will not be identical (Binney & Tremaine, 2008). Therefore, the system will have a natural frequency which is represented by ω_0 in equation (1). The parameter, α, in the second term appears because of the various tidal forces (other than due to GWs) . The α measures the resistance to change. The resistance can be viewed as any drag force such as dynamical friction. This is more pronounced in a closely packed system of stars like a globular cluster. Depending on where the binary system is in the potential well of the external system of stars, ω_0 can have a huge range of values.

The differential equation (1) has the general form of a driven harmonic oscillator. For simplicity, let us consider a sinusoidal gravitational wave with only h_+ polarization interacting with the edge-on binary system. Furthermore, we shall consider a GW that interacts with the binary system face-on; that is, the GW propagates perpendicularly to the line-of-sight. The λ term in equation (1) is due to the incoming GW wave. The incoming GW propagate in the z-direction of a TT-coordinate system. Without any loss of generality, we can consider a GW having only a single polarization, h_+ (i.e., $h_\times = 0$). Then in this case the tidal acceleration matrix takes a simpler form and equation (1) reduces to

$$\ddot{\xi} + \alpha\dot{\xi} + \omega_0^2 \xi = -\frac{1}{2}\omega^2 d\, \Re[e^{-iwt} h_+]. \tag{2}$$

The particular or the steady-state solution of the above equation is

$$\xi = \Re\left[\frac{d\omega^2 e^{-i\omega t} h_+}{2(\omega^2 - \omega_0^2 + i\omega\alpha)}\right]. \tag{3}$$

2.2. Sufficiently Isolated Binary System

Let us consider a sufficiently isolated binary system over which the tidal forces are at a minimum. In such a realistic case, damping time $(1/\alpha)$ is much larger than the natural period $(1/\omega_0)$ of the system. In this case, equation (3) reduces to

$$\xi = \frac{1}{2} d h_+ \,\cos\omega t. \tag{4}$$

Hence, while the interaction is taking place, the system of stars will oscillate with a certain frequency in the radial direction. The radial velocity, v, of a star is

$$v = \dot{\xi} = -\frac{1}{2} d\omega h_+ \,\sin\omega t. \tag{5}$$

The maximum value of the radial velocity is

$$v_{\max} = \frac{1}{2} d\omega h_+ = \pi d f h_+, \tag{6}$$

where f is the frequency of the gravitational wave having the strain $h_+ \propto r^{-1}$. For $v_{\max} \ll c$, the induced maximum radial Doppler shift, $z_{\max}$, is given by

$$z_{\max} \approx \frac{v_{\max}}{c} = \frac{\pi h_+ f d}{c}. \tag{7}$$

Thus,

$$h_+ = \frac{c\, z_{\max}}{\pi f d}. \tag{8}$$

Using the current Doppler shift detectability limit we may write the above equation as,

$$h_+ = \frac{c\, 10^{-8}}{\pi f d} \approx 0.318 \times 10^{-8}\, \frac{1}{f(d/c)}. \tag{9}$$

Hence, the sensitivity of the method depends on the frequency, f, of the gravitational wave and the separation distance, d, between the two stars. Fig. 3 shows the sensitivity of the method as a function of GW frequency for various separations.

2.3. Binary System in a Potential

In practice, binary star systems are not isolated. They have their own mutual gravity (stars cannot be treated as free particles) and lie in an external potential well (such as the galactic potential well) and also may be affected by tidal forces between the two stars and the other stars nearby. In this case, α and ω_0 cannot be

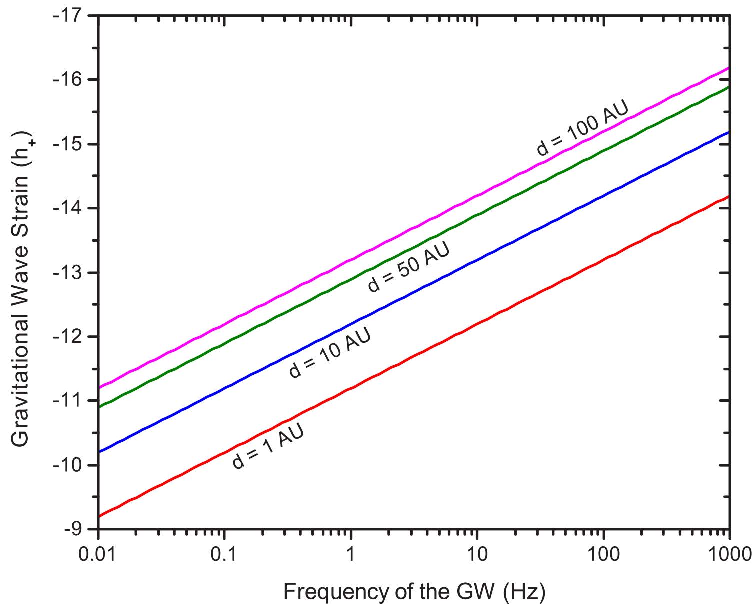

Fig. 3. The sensitivity of the method with the current Doppler detectability limit of z = 10^{-8}. Various lines represent sensitivity curves for stellar binaries with d = 1, 10, 50 and 100 AU.

neglected. In order to consider these effects let us recast equation (3) as follows:

$$\xi = \Re[e^{-i\omega t}(a - ib)] \tag{10}$$

Here,

$$a = \Gamma(\omega^2 - \omega_0^2), \;\; b = \Gamma\omega\alpha, \tag{11}$$

and

$$\Gamma = \frac{1}{2}\frac{d\omega^2 h_+}{(\omega^2 - \omega_0^2)^2 + \omega^2\alpha^2}. \tag{12}$$

Equation (10) can be further simplified by defining, $\tan\beta = b/a$ as,

$$\xi = \sqrt{a^2 + b^2}\,\cos(\omega t + \beta). \tag{13}$$

Hence, the stars oscillate with a phase difference, β, with respect to the GW. The radial velocity, v, of a star is

$$v = \dot{\xi} = -\omega\sqrt{a^2 + b^2}\,\sin(\omega t + \beta). \tag{14}$$

The magnitude of the maximum radial velocity is

$$v_{\max} = \omega\sqrt{a^2 + b^2} = \frac{d\omega^3 h_+}{2\sqrt{(\omega^2 - \omega_0^2)^2 + \omega^2\alpha^2}}. \tag{15}$$

It is clear from the foregoing that $v_{\max}$ becomes maximum near the resonance, $\omega \approx \omega_0$. If the damping time $\propto \frac{1}{\alpha} \gg \frac{1}{\omega_0}$ (natural period of oscillations of the system) then from equation (15),

$$v_{\max} = \frac{d\omega^3 h_+}{2(\omega^2 - \omega_0^2)}. \tag{16}$$

Near resonance, taking $\Delta\omega = \omega - \omega_0$, we get,

$$v_{\max} \approx \frac{d\omega^2 h_+}{4\Delta\omega} = \frac{d\pi f^2 h_+}{2\Delta f}. \tag{17}$$

Hence near the resonance, the detector has an enhanced response which absorbs large amount of energy from the GW. Thus, at near resonance, it is possible that gravitational waves from a nearby source may unbind the stars of a closed binary system and even potentially eject them (see §3.2).

3. Discussion

3.1. Potential GW Detector Systems

Currently, the distance to the furthest exoplanet detected using Doppler spectroscopy is $\sim$ 2000 parsecs (Lovis & Mayor, 2007). Thus, in the detectable volume of $\sim\ 10^{10}$ cubic parsecs, there are roughly about $\sim 10^9$ stars (assuming the solar neighborhood volume density of 0.033 solar masses per cubic parsecs; Binney & Tremaine (2008)). However, we note that the accurate number of stars will be much smaller than this due to the exponential drop of stellar density in the direction perpendicular to the plane of the Galaxy. We also know that about one-third of all main-sequence stars in the Galactic disk belong to either binary or multiple star systems (Lada, 2006). If we assume that the binary systems are oriented randomly, then by simple geometric arguments about 20% of the star systems are near edge-on binary systems (considering only systems with less than 10 degree orbital angles with line-of-sight). Hence, there are roughly about $\sim\ 10^7$ edge-on binary systems in the solar neighborhood. However, only a very small fraction of these edge-on binaries will be in the vicinity of potential GW sources. The most promising would be edge-on binary systems within globular clusters or open clusters.

3.2. GW Interactions as a Potential Source of High Velocity Stars?

There is a very interesting implication that may arise from the interaction of a GW with a binary star system. According to equation (17), when the frequency of the GW is in the neighborhood of the resonance frequency of the binary system, $v_{\max}$ becomes very large. This implies that the GW may potentially unbind the binary system.

It is also important to investigate whether the total energy budget of a typical GW source is large enough to unbind a stellar system. For example, if we consider a pair of orbiting 3 solar mass black holes, their GW luminosity (or energy emitted per second) will increase as they spiral towards each other as shown in Fig. 4 (Hobson et al., 2006). In the same figure, we

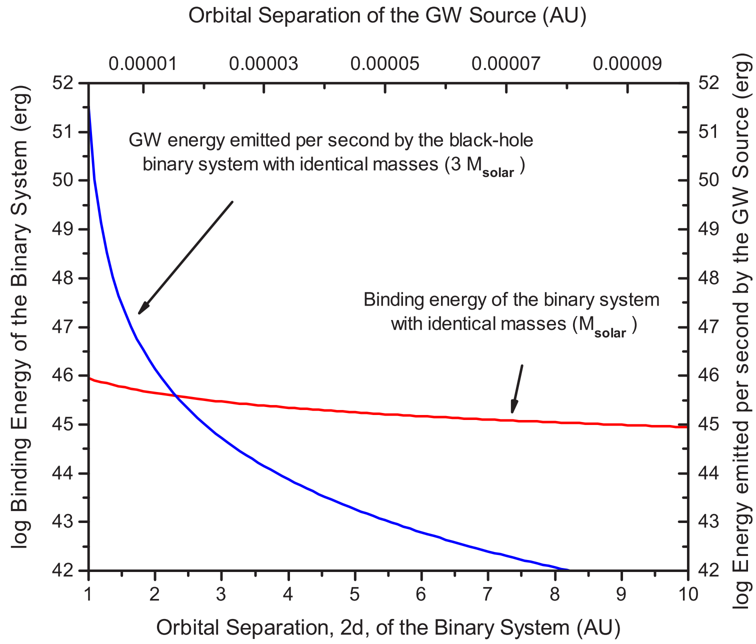

Fig. 4. Comparison of the GW energy emitted by a $3M_{\odot}$ black hole pair for a time duration of one second and the binding energy of one solar mass binary star system.

have shown the binding energy of a binary star system with identical masses of one solar mass (red curve). The GW energy flux emitted from such a system is highly anisotropic and drops off as the square of the distance far away from the source. In the plot, it is clear that that there is a region where GW energy emitted per second far exceeds the binding energy of the binary system. For this particular GW source, the duration in which the luminosity is greater than the binding energy lasts for more than 8 hours. This may give adequate time for a binary star system of suitable distance and separation to absorb enough energy at a frequency near resonance to unbind itself. Hence, in principle, a GW source which emits its energy anisotropically has more than enough energy to unbind a stellar system in its proximity. However, it is important to note here that if GW source emit its energy isotropically in all directions, then it is unlikely that the stellar binary will be able to absorb enough energy to unbind itself.

The abundance of runaway and hyper-velocity stars in the halos of galaxies have puzzled astronomers (Bromley et al., 2009; Lu et al., 2010). One of the leading sources for these high velocity stars is the scenario deriving from supernova explosions. Another method would be the ejection of star systems that approach too close to the super-massive black-holes at the center of galaxies. Gravitational waves hitting binary stars under certain conditions could provide another potential mechanism. One important distinction worth noticing here is that a single GW source may in principle eject multiple stars.

4. Conclusion

In this paper, we have explored the possibility of using Doppler effects caused by a passing gravitational wave on an edge-on binary star system as a direct gravitational wave detection method. In our simple analysis, we find that with current instrumentation we might be able to detect gravity waves with strains in the range $h \sim 10^{-11}$ to $h \sim 10^{-13}$ depending on the separation of the binary system and the frequency of the gravitational wave in question (in this case at 1 Hz). Considering the abundance of binary systems in galaxies and in globular clusters alike, we propose that such systems could be used as potential GW probes in our own galaxy. As the spectroscopic instrumentation is improving steadily, Dopplershift limits may increase substantially to probe GW sources in the Galaxy, nearby galaxies and stellar systems in the future.

References

Binney, J., & Tremaine, S. 2008, Galactic Dynamics: Second Edition, by James Binney and Scott Tremaine. ISBN 978-0-691-13026-2 (HB). Published by Princeton University Press, Princeton, NJ USA, 2008.,

Bromley, B. C., Kenyon, S. J., Brown, W. R., & Geller, M. J. 2009, *ApJ.*, 706, 925

Einstein, A. 1916, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 688-696., 688

Freise, A., & Strain, K. 2010, Living Reviews in Relativity, 13, 1

Gwinn, C. R., Eubanks, T. M., Pyne, T., Birkinshaw, M., & Matsakis, D. N. 1997, *ApJ.*, 485, 87

Hobson, M. P., Efstathiou, G. P., & Lasenby, A. N. 2006, General Relativity, by M. P. Hobson, G. P. Efstathiou and A. N. Lasenby, pp. 590. Cambridge University Press, February 2006. ISBN-10: 0521829518. ISBN-13: 9780521829519. LCCN: QC173.6 .H63 2006,

Lada, C. J. 2006, *ApJ. Lett.*, 640, L63

Lovis, C., & Mayor, M. 2007, *A.& A.*, 472, 657

Lu, Y., Zhang, F., & Yu, Q. 2010, *ApJ.*, 709, 1356

Mayor, M., et al. 2009, *A.& A.*, 493, 639

Ni, W. T. 2010, arXiv:1003.3899

Pyne, T., Gwinn, C. R., Birkinshaw, M., Eubanks, T. M., & Matsakis, D. N. 1996, *ApJ.*, 465, 566

Taylor, J. H., & Weisberg, J. M. 1989, *ApJ.*, 345, 434